Math 110

Winter 2021

Lecture 12

Binomial Prob. Dist:

Ex. Consider a binomial Prob. dist with $n=250$ and $p=.8$

1) $q=1-p=\boxed{.2}$

2) $\mu=np=\boxed{200}$

3) $\sigma^2=npq=\boxed{40}$

4) $\sigma=\sqrt{\sigma^2}=\sqrt{40}=\boxed{6.325}$

Round μ & σ to a whole #, then find

$\boxed{\mu=200}$ $\boxed{\sigma=6}$

5) 68% Range

$\mu\pm\sigma$

$=200\pm6 \Rightarrow \boxed{194 \text{ to } 206}$

6) Usual Range

95% Range

$\mu\pm2\sigma$

$=200\pm2(6)$

$=200\pm12$

$\Rightarrow \boxed{188 \text{ to } 212}$

Let x be # of successes, find

7) $P(x=185)$

$=\text{binompdf}(250, .8, 185)$

$=\boxed{.004}$

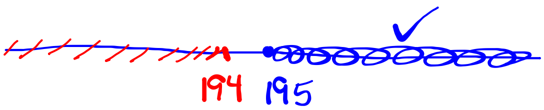
8) $P(x\le210)$

$=\text{binomcdf}(250, .8, 210)$

$=\boxed{.955}$

9) $P(x \ge 195)$

$= 1 - P(x \le 194) = 1 - \text{binomcdf}(250, .8, 194)$



$= \boxed{.809}$

10) $P(190 \le x \le 210)$

Reduce by 1

$= \text{binomcdf}(250, .8, 210)$

$- \text{binomcdf}(250, .8, 189)$

$= \boxed{.904}$

180 voters were randomly selected. Prob. that each voter supports tougher gun law is .55.

1) $n = 180$ 2) $p = .55$ 3) $q = 1 - p = .45$

4) $\mu = np = \boxed{99}$ 5) $\sigma^2 = npq = \boxed{44.55}$ 6) $\sigma = \sqrt{\sigma^2} = \boxed{6.675}$

Round μ & σ to a whole #, find $\boxed{\mu = 99}$ $\boxed{\sigma = 7}$

7) 68% Range

$\mu \pm \sigma = 99 \pm 7 \Rightarrow \boxed{92 \text{ to } 106}$

8) $\underbrace{\text{Usual Range}}$ 95% Range

$\mu \pm 2\sigma = 99 \pm 2(7)$

$= \boxed{85 \text{ to } 113}$

9) P(exactly 100 support such law) $= P(x = 100)$

$= \text{binompdf}(180, .55, 100) = \boxed{.059}$

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10) P($\underbrace{\text{fewer than 110}}$ support such law)

$= P(x < 110) = P(x \leq 109)$

$= \text{binomcdf}(180, .55, 109)$

$= \boxed{.943}$

11) P($\underline{\text{more than 90}}$ support such law)

$P(x > 90) = P(x \geq 91) = 1 - P(x \leq 90)$

$= 1 - \text{binomcdf}(180, .55, 90)$

90 91

$= \boxed{.898}$

12) P($\underbrace{\text{between 90 and 110}}$, $\underline{\text{inclusive}}$, support such law)

$P(90 \leq x \leq 110) = \text{binomcdf}(180, .55, 110) -$

Reduce by 1

$\text{binomcdf}(180, .55, 89) = \boxed{.881}$

$\cancel{90 \geq x \geq 110}$

Geometric Prob. Dist. SG 18

1) Independent events, but # of trials n is not fixed.

2) Trial repeats x times until success happens.

3) Prob. of success is p, remains the same for all trials.

4) x is the number that first success happens

$$P(x) = p \cdot (q)^{x-1}, \quad q = 1 - p$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{q}{p^2}, \quad \sigma = \sqrt{\sigma^2}$$

Consider a geometric Prob. dist with $p = .4$

1) $q = 1 - p$
$= \boxed{.6}$

2) $\mu = \frac{1}{p} = \frac{1}{.4}$
$= \boxed{2.5}$

3) $\sigma^2 = \frac{q}{p^2} = \frac{.6}{.4^2}$
$= \boxed{3.75}$

4) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{3.75} = \boxed{1.936}$

5) $P(x = 3)$ $P(x) = p \cdot q^{x-1}$
$= .4(.6)^{3-1} = .4(.6)^2$
$= \boxed{.144}$

using TI
2nd VARS ↓ ...
geomet pdf$(.4, 3)$
$\uparrow$ p x

6) $P(x \leq 4) = P(x=4) + P(x=3) + P(x=2) + P(x=1)$
$= \text{geometcdf}(.4, 4) = \boxed{.870}$

7) $P(x \geq 5) = P(x=5) + P(x=6) + P(x=7) + \cdots$
$= 1 - P(x \leq 4)$
$= 1 - \text{geometcdf}(.4, 4) = \boxed{.13}$

4 5

Prob. You make a basketball shot is .3

1) $p = .3$

2) $q = .7$

3) $\mu = \frac{1}{p} = \frac{1}{.3} = \frac{10}{3}$
$= 3.333$

4) $\sigma^2 = \frac{q}{p^2} = \frac{.7}{.3^2} = \boxed{7.778}$

5) $\sigma = \sqrt{\sigma^2}$
$= 2.789$

6) P(make first basket in 5th trial)
$= P(x=5) = \text{geometpdf}(.3, 5) = \boxed{.072}$

7) P(make first basket in 5th trial or below)
$= P(x \leq 5) = \text{geometcdf}(.3, 5) = \boxed{.832}$

8) P(make first basket after the 5th trial)
$= P(x \geq 6) = 1 - P(x \leq 5)$
$= 1 - \text{geometcdf}(.3, 5) = \boxed{.168}$

Poisson Prob. Dist.

1) This prob. takes Place in a fixed interval.

2) Prob. of two or more Successes in a Small interval $= 0$

3) μ is the mean, $\sigma^2 = \mu$

4) $P(x) = \frac{\mu^x}{x!} e^{-\mu}$ $\quad e \approx 2.7183$

$P(x = a) =$ Poisson PdF(μ, a)

$P(x \leq a) =$ Poisson cdF(μ, a)

$P(x \geq a) = 1 -$ PoissoncdF$(\mu, a-1)$

Alex gets 24 Calls for repairs in 8-hr shift.

So in average, he gets 3 Calls $\underbrace{\text{per hour.}}_{\text{Interval}}$

$\mu = 3$

$\lambda = \mu = 3$

$P(\text{he gets 2 Calls}) = P(x = 2)$

$=$ Poisson PdF$(3, 2) = \boxed{.224}$

$P(\text{he gets at most 5 Calls}) = P(x \leq 5)$

$=$ PoissoncdF$(3, 5)$

$= \boxed{.916}$

$P(\text{he gets at least 4 Calls}) =$

$P(x \geq 4) = 1 - P(x \leq 3) = 1 -$ PoissoncdF$(3, 3)$

$= \boxed{.353}$

Choose 3 numbers from 1 to 25.
House draws 3 numbers, as winning numbers.
You have to pay \$1 to Play.
If You have 3 winning # ⇒ You get \$100
If You have 2 " " ⇒ You get \$10
If You have 1 " " ⇒ You get \$1
otherwise You get nothing. Find expected Value Per ticket Sold.

win. #	\$Net gain	P(\$Net gain)
3	1 − 100	$\frac{{}_3C_3 \cdot {}_{22}C_0}{{}_{25}C_3} = \frac{1}{2300}$
2	1 − 10	$\frac{{}_3C_2 \cdot {}_{22}C_1}{{}_{25}C_3} = \frac{66}{2300}$
1	1 − 1	$\frac{{}_3C_1 \cdot {}_{22}C_2}{{}_{25}C_3} = \frac{693}{2300}$
0	1 − 0	$\frac{{}_3C_0 \cdot {}_{22}C_3}{{}_{25}C_3} = \frac{1540}{2300}$

\$Net gain → L1 , P(\$Net gain) → L2

Use L1 & L2 to find E.V. = $\mu = \bar{x}$ .368

E.V. ≈ 37¢/ticket

Class QZ 7

Given $P(A) = .7$ $P(B) = .5$ $P(A \text{ and } B) = .4$

1) Venn Diagram

2) $P(A \text{ or } B)$

3) $P(A \mid B)$